GOSFORD HIGH SCHOOL
MATHEMATICS
HSC Assessment Task 1:
December 2010

Time: 60 minutes.

Part A

Question 1.

a) Find the value of:

i) $\lim_{x \longrightarrow 3} \frac{x^3 - 27}{x + 3}$ **(1)**

ii) $\lim_{x \longrightarrow \infty} \frac{7x \ + 3}{3x \ + 2}$ **(2)**

Question 2.

Differentiate.

a) $y = 5x^3 - 3x^2 + 5$ **(2)**

b) $y = x\left(\sqrt[3]{x^2}\right)$ **(2)**

c) $f(x) = \frac{2x+1}{x^2 - 2}$ **(2)**

Question 3.

Find the second derivative of the following.

a) $f(x) = 4x^3 + \frac{2}{x} + 1$ **(2)**

b) $y = \left(3 + 2x^2\right)^3$ **(3)**

Question 4.

a) For what values of x is the curve $y = x^3 + 3x^2 - 8$ increasing? **(3)**

b) At what point on the curve $y = x^2 + 5x + 6$ is the gradient of the tangent equal to 1? **(2)**

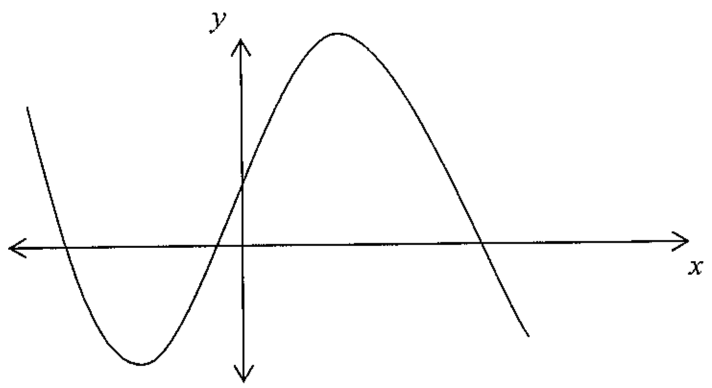
c) What is the gradient of the normal to the curve $y = x^3 - 3x^2 - 9x + 1$ at the point $(1, -10)$? **(2)**

Question 5.

a) For the curve $y = ax^2 + bx + c$ where a, b and c are constants, it is given that $y = 1$ and $\frac{dy}{dx} = 1$, when $x = 1$. Show that $a = c$. **(3)**

b) In the interval $a \le x \le b$ the curve $y = f(x)$ has: $y < 0$, $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$. Sketch a possible graph of $y = f(x)$. **(2)**

c)



The graph shows $y = f(x)$. Copy the graph onto your answer sheet and on the same axes, draw the graph of $y = f'(x)$. **(2)**

Question 6.

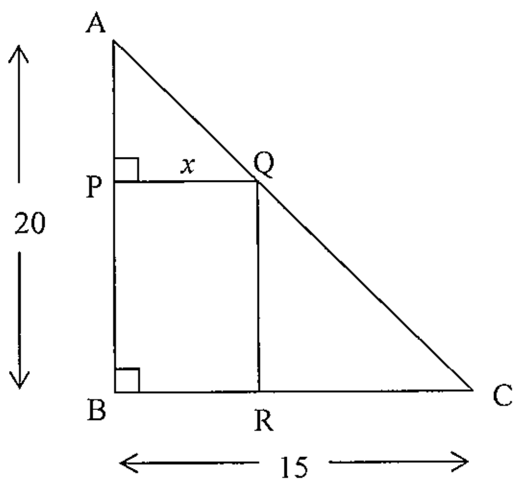
a) Given the curve $y = x^3 - 3x^2 - 9x + 1$

i) Find the coordinates of any stationary points and determine their nature. **(3)**

ii) What are the coordinates of any points of inflexion? **(2)**

iii) Sketch the curve. **(1)**

Question 7.



In triangle ABC, AB = 20 *cm*, BC = 15 *cm* and angle ABC is a right angle. BPQR is a rectangle inscribed in triangle ABC as in the figure with PQ $= x$ *cm*

a) Given that AP is $\frac{4x}{3}$ *cm,* show that the area (A) of the rectangle BPQR is given by

$$A = x\left(20 - \frac{4x}{3}\right)$$ **(1)**

b) Find the maximum area of the rectangle BPQR. **(4)**

PART B

Question 1.

a) Write the locus of all points 3 units from the y axis. **(1)**

b) Write the centre and radius for the circle $x^2 - 6x + y^2 + 2y + 6 = 0$. **(2)**

c) A and B are the points $(2,4)$ and $(4,2)$ respectively. Find the locus of the point $P(x, y)$ which moves so that the gradient of PA is twice the gradient of PB. **(3)**

Question 2.

a) A parabola has the equation $(x-2)^2 = 6(y+4)$, find:

i) Coordinates of the vertex. **(1)**
ii) Coordinates of the focus. **(1)**
iii) Equation of the directrix. **(1)**

b) Write the equation of the parabola with focus $(4,0)$ and directrix $x+4=0$. **(1)**

c) A parabola has the equation $y = \frac{1}{2}x^2 - 3x + 1$. Express this equation in the form $(x-h)^2 = 4a(y-k)$ where a, h and k are constants. **(2)**

d) $P(x, y)$ is a point that moves so that its distance from $A(3,1)$ is always equal to its distance from the line $y = -5$. Derive the equation of the locus of P. **(2)**

Mathematics : assessment task 1, Dec 2010 - SOLUTIONS (Mark 2)

Q1) a) i) $\lim_{x \to 3} \frac{x^3-27}{x+3}$

$= \frac{0}{6}$

$= 0$

ii) $\lim_{x \to \infty} \frac{7x+3}{3x+2}$

$\lim_{x \to \infty} \frac{\frac{7x}{x} + \frac{3}{x}}{\frac{3x}{x} + \frac{2}{x}}$

$\lim_{x \to \infty} \frac{7 + \frac{3}{x}}{3 + \frac{2}{x}}$

$= \frac{7+0}{3+0}$

$= \frac{7}{3}$

Q2) a) $y = 5x^3 - 3x^2 + 5$

$y' = 15x^2 - 6x$

b) $y = x(\sqrt[3]{x^2})$

$= x \cdot x^{2/3}$

$= x^{5/3}$

$y' = \frac{5}{3} x^{2/3}$

$= \frac{5\sqrt[3]{x^2}}{3}$

c) $f(x) = \frac{2x+1}{x^2-2}$

$f'(x) = \frac{(x^2-2)\times 2 - 2x(2x+1)}{(x^2-2)^2}$

$= \frac{2x^2-4-4x^2-2x}{(x^2-2)^2}$

$= \frac{-2x^2-2x-4}{(x^2-2)^2}$

Q3) a) $f(x) = 4x^3 + \frac{2}{x} + 1$

$= 4x^3 + 2x^{-1} + 1$

$F'(x) = 12x^2 - 2x^{-2}$

$f''(x) = 24x + 4x^{-3}$

$f''(x) = 24x + \frac{4}{x^3}$

b) $y = (3+2x^2)^3$

$y' = 3.(3+2x^2)^2.4x$

$y' = 12x(3+2x^2)^2$

$y'' = 12x.2(3+2x^2).4x + 12(3+2x^2)^2$

$= 96x^2(3+2x^2) + 12(3+2x^2)^2$

$= 12(3+2x^2)(8x^2+3+2x^2)$

$= 12(3+2x^2)(10x^2+3)$

Q4) a) $y = x^3 + 3x^2 - 8$

increasing when $y' > 0$

$y' = 3x^2 + 6x$

increasing $3x^2 + 6x > 0$

$3x(x+2) > 0$

$-2 \quad 0$

test $x = 1$: $3 \times 3 > 0$ T.

$\therefore x < -2$ or $0 < x$

b) $y = x^2 + 5x + 6$

$y' = 2x + 5$

$\therefore 2x + 5 = 1$

$2x = -4$

$x = -2.$

$y = 4 - 10 + 6$

$= 0$

$\therefore$ pt $(-2, 0)$

c) $y = x^3 - 3x^2 - 9x + 1$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

at $x = 1$, $\frac{dy}{dx} = 3 - 6 - 9$

$= -12$

$\therefore$ gradient of normal $= \frac{1}{12}$

Q5) a) $y = ax^2 + bx + c$

$y' = 2ax + b.$

$x = 1$: $y = 1$, $\frac{dy}{dx} = 1.$

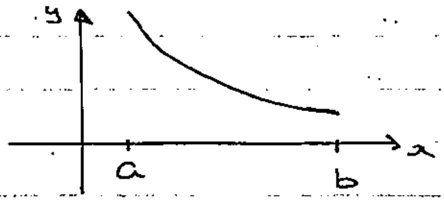
$\therefore 1 = a + b + c$ ---(1)

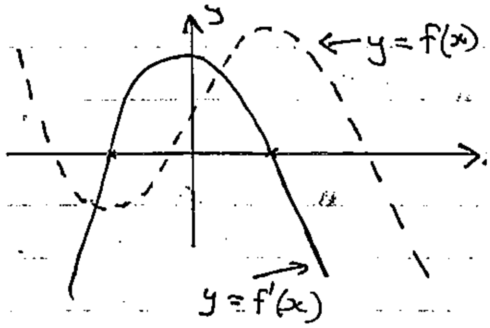
$1 = 2a + b$ ---(2)

(2)-(1) $0 = a - c$

$\therefore a = c.$

b) $y < 0$, $\frac{dy}{dx} < 0$, $\frac{d^2y}{dx^2} > 0$





Q6) i) $y = x^3 - 3x^2 - 9x + 1$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

$\frac{d^2y}{dx^2} = 6x - 6$

stationary pts. $\frac{dy}{dx} = 0.$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\left.\begin{matrix} x = 3 \\ y = -26 \end{matrix}\right\}$ or $\left.\begin{matrix} x = -1 \\ y = 6 \end{matrix}\right\}$

$f''(3) > 0$ $\qquad$ $f''(-1) < 0$

$\therefore (3, -26)$ min $\qquad$ $\therefore (-1, 6)$ max

(i) pts. of inflexion
$\frac{d^2y}{dx^2} = 0$, change in concavity.

$6x - 6 = 0$

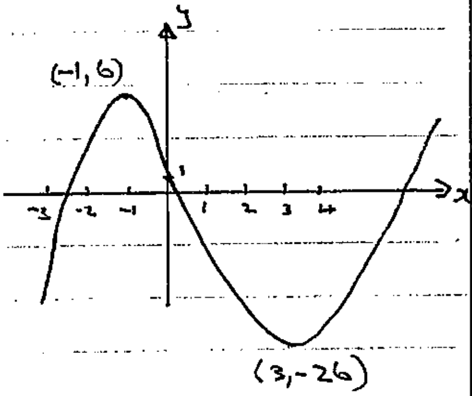
$x = 1$

$y = -10$

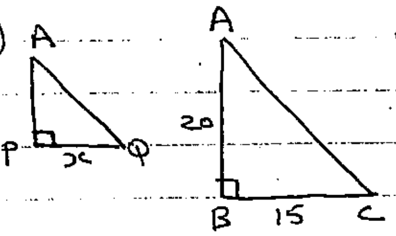
$f''(0) < 0, \quad f''(2) > 0$

∴ change in concavity

∴ $(1, -10)$ pt. of inflexion



Q7) a)



$\triangle APQ \; ||| \; \triangle ABC$ (AAA)

∴ $\frac{AP}{20} = \frac{x}{15}$

$AP = \frac{20x}{15}$

$= \frac{4x}{3}$

b) $PB = 20 - AP$

$= 20 - \frac{4x}{3}$

Area rectangle $= L \times b$

$A = x\left(20 - \frac{4x}{3}\right)$

c) $A = x\left(20 - \frac{4x}{3}\right)$

$= 20x - \frac{4x^2}{3}$

$\frac{dA}{dx} = 20 - \frac{8x}{3}$

$\frac{d^2A}{dx^2} = -\frac{8}{3}$

Max when $\frac{dA}{dx} = 0$

$20 - \frac{8x}{3} = 0$

$20 = \frac{8x}{3}$

$8x = 60$

$x = 7\frac{1}{2}$

as $\frac{d^2A}{dx^2} < 0$ ∴ max.

∴ max area

$= 20 \times 7\frac{1}{2} - \frac{4(7\frac{1}{2})^2}{3}$

$= 75 \text{ cm}^2$.

PART B

Q1) a) $x = 3$, or $x = -3$ (1)

b) $x^2 - 6x + y^2 + 2y + 6 = 0$

$x^2 - 6x + 9 + y^2 + 2y + 1 = -6 + 9 + 1$

$(x-3)^2 + (y+1)^2 = 4$

centre $(3, -1)$ radius $= 2$.

1 1

c) $m_{PA} = \frac{y-4}{x-2}$, $m_{PB} = \frac{y-2}{x-4}$

$m_{PA} = 2m_{PB}$

$\frac{y-4}{x-2} = \frac{2(y-2)}{x-4}$

$(x-4)(y-4) = (x-2)(2y-4)$

$xy - 4x - 4y + 16 = 2xy - 4x - 4y + 8$

$xy = 8$.

But $x \neq 2$ or 4 (3)

Q2) a) $(x-2)^2 = 6(y+4)$

i) $(2, -4)$

ii) $(2, -2\frac{1}{2})$

iii) $y = -5\frac{1}{2}$.

b) $y^2 = 16x$

c) $y = \frac{1}{2}x^2 - 3x + 1$

$2y = x^2 - 6x + 2$

$2y - 2 = x^2 - 6x$

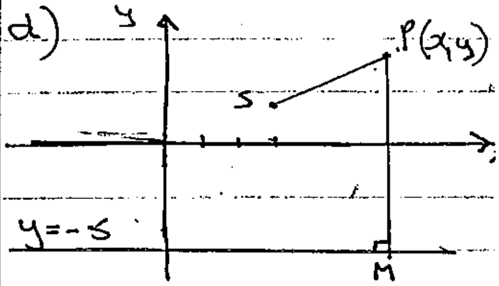
$2y - 2 + 9 = x^2 - 6x + 9$

$2y + 7 = (x-3)^2$

$(x-3)^2 = 2(y + 3\frac{1}{2})$

$(x-3)^2 = 4 \times \frac{1}{2}(y + 7/2)$

d)



$PS = PM$

$\sqrt{(x-3)^2 + (y-1)^2} = y + 5$

$x^2 - 6x + 9 + y^2 - 2y + 1 = y^2 + 10y + 25$

$x^2 - 6x + 10 = 12y + 25$

$x^2 - 6x + 9 + 1 = 12y + 25$

$(x-3)^2 = 12y + 24$

$(x-3)^2 = 12(y+2)$

(2)

REVISION FOR ASSESSMENT TASK 1

PART A

Question 1.

a) Find the value of:

i) $\lim\limits_{x \to 2} \frac{x^2+x-1}{x-1}$ ii) $\lim\limits_{x \to 3} \frac{x^3-27}{x-3}$ iii) $\lim\limits_{x \to \infty} \frac{x^2+3x}{3x^2+2}$

b) Differentiate from first principals $3x^2-5x+1$

Question 2.

Differentiate.

a) $y=x^3+3x^2-1$ b) $y=x\sqrt{x}$ c) $y=\left(x^2+3x\right)^4$

d) $f(x)=\frac{x^2+1}{x-2}$ e) $f(x)=x\sqrt{x^2+1}$ f) $\frac{3}{1-x\sqrt{2}}$ g) π^3

h) a^2x-ax^2 where a is a constant.

Question 3.
Find the second derivative of the following.

a) $f(x)=4x^3-3x^2+2x+1$ b) $y=\left(3x^2+2\right)^3$

Question 4.

a) For what values of x is the curve $y=x^2+2x-8$ increasing ?

b) For what values of x is the curve $y=x^3+x^2+x$ concave down ?

c) At what point on the curve $y=x^2-5x+6$ is the gradient of the tangent equal to 1?

d) At what point(s) on the curve $y=x^3-3x^2-9x+1$ is the gradient of the normal equal to $\frac{1}{9}$?

e) Where does the tangent to the curve $y=x^3$ slope at $45°$

Question 5.

a) Find a, b and c if the curve $y=x^3+ax^2-bx+c$ has an x- intercept at $x=1$, a stationary point at $x=-2$ and a point of inflexion at $x=-\frac{1}{2}$.

b) Differentiate $y=a(x+b)^2-8$, then find a and b if the parabola has a tangent $y=2x$ at the point $(4,8)$

c) If $y=\sqrt{2x+4}$, show $y\frac{dy}{dx}$ is a constant.

d) If $f(x)=x^3-2x+1$ find $f(-1)-f'(-1)$

e) Given the function $f(x)=x^2-3x+3$ find the values of b for which $f(b)=f'(b)$

Question 6.
The tangent to the curve $y=3x^3-8x^2$ at the point of contact, P(2,-8) cuts the x- axis at A and the normal to the curve at the same point of contact cuts the y- axis at B.

a) find the equation of the tangent at P.

b) find the equation of the normal at P.

c) find the coordinates of A and B.

d) Find the length of the interval AB.

Question 7.

a) Consider the curve given by $y=1+3x-x^3$ for $-2 \le x \le 3$.

i) Find the turning points and determine their nature.

ii) What are the coordinates of any points of inflexion ?

iii) Sketch the curve for $-2 \le x \le 3$.

iv) What are the maximum and minimum values of y for $-2 \le x \le 3$.

b) Consider the curve given by $y=x+\frac{1}{x}$

i) State the equation of any vertical asymptotes.

ii) Describe the behaviour of the function as $x \to \infty$

iii) Find the turning points and determine their nature.

iv) Sketch the curve.

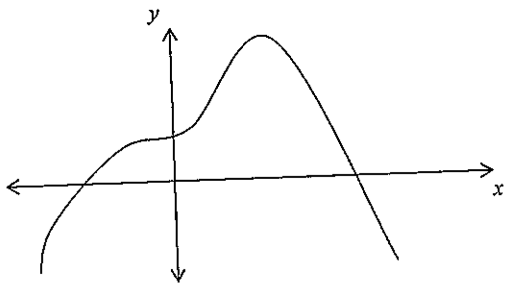
Question 8.
On separate number planes draw neat sketches to illustrate the required graphs below.

a) For this graph $\frac{dy}{dx}=3$ for all values of x. The graph passes through the point $(1,2)$.

b) For this graph $\frac{dy}{dx}<0$ for $x<2$: at $x=2$ $\frac{dy}{dx}=0$: $\frac{dy}{dx}>0$ for $x>2$. The graph passes through the point $(2,-1)$.

c) For this graph $\frac{dy}{dx}$ does not exist at all. The graph passes through the point $(1,-1)$.

Question 9.

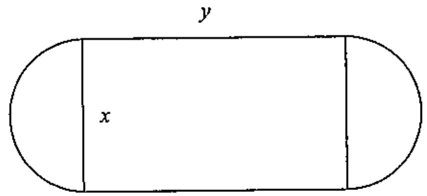


The graph shows $y=f(x)$.

a) Copy the graph onto your answer sheet.

b) On the same axes, draw the graph of $y=f'(x)$

Question 10.
A railway enthusiast designs a miniature railway of length 1000 metres. The route consists of two semicircles at opposite ends of a rectangle.



a) If the rectangle has length y metres and its width is x metres, show that:
$y=500-\frac{\pi x}{2}$.

b) show that the area, A, enclosed by the railway track is given by $A=\frac{2000x-\pi x^2}{4}$

c) Find the maximum area, to the nearest hectare, enclosed by the railway track.

PART B

Question 1
a) Write the centre and radius for the following circles.
i) $x^2+y^2=16$

ii) $(x+1)^2+(y-2)^2=12$

iii) $x^2-4x+y^2+6y+6=0$.

b) Write the locus of all points equidistant from the x and y axes.

c) Find the locus of the point $P(x,y)$ which moves so that its distance from the point $A(4,0)$ is always twice its distance from the point $B(1,0)$.

d) For the points $A(3,0)$ and $B(-3,0)$ find the locus of the point $P(x,y)$ which moves so that PA is always perpendicular to PB.

Question 2.

a) For the parabola $x^2 = -16y$ find the:

i) Focal length.
ii) Coordinates of the focus.
iii) Equation of the directrix.

b) A parabola has the equation $x^2 + 6x - 33 = 12y$. Find:

i) Coordinates of the vertex.
ii) Coordinates of the focus.
iii) Equation of the directrix.

c) A parabola has the equation $2y^2 = 4x + 8$.

i) Express this equation in the form $(y - y_1)^2 = 4a(x - x_1)$
ii) Draw a neat sketch of the parabola indicating the coordinates of the focus, the equation of the directrix and the coordinates of all points of intersection of the parabola and the coordinate axes.

d) $P(x.y)$ is a point that moves so that its distance from $A(2,1)$ is always equal to its distance from the line $y = -1$. Show that the equation of the locus of P is $(x-2)^2 = 4y$.

e) Determine the equation of the locus of the point $P(x, y)$ which is equidistant from the y axis and the point $(1,0)$.

f) Write the equation of the following parabolas.

i) Focus $(0,5)$ directrix $y + 5 = 0$.
ii) Focus $(2,0)$ directrix $x + 2 = 0$.

g) Find the equation of the parabola with its vertex at $(1,4)$, axis parallel to the y axis and passing through the point $(3,5)$.

h)

i) Sketch $y = x^2$ and $x = y^2$ on the same number plane and show that they intersect at $O(0,0)$ and $A(1,1)$.

iii) Using calculus, find the equation of the tangent to $y = x^2$ at A and its y intercept P.

iv) By differentiating $y = \sqrt{x}$ find Q where the tangent to $x = y^2$ at A crosses the x axis.

v) Show that $AP = AQ = \sqrt{5}$ and find the area of triangle POQ.

SOLUTIONS: REVISION FOR ASSESSMENT TASK 1.

Q1) a) i) $\lim\limits_{x\to 2} \dfrac{x^2+x-1}{x-1}$

$= \dfrac{4+2-1}{2-1}$

$= 5$

ii) $\lim\limits_{x\to 3} \dfrac{x^3-27}{x-3}$

$= \lim\limits_{x\to 3} \dfrac{(x-3)(x^2+3x+9)}{x-3}$

$= \lim\limits_{x\to 3} x^2+3x+9$

$= 27$

iii) $\lim\limits_{x\to\infty} \dfrac{x^2+3x}{3x^2+2}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{x^2}{x^2}+\frac{3x}{x^2}}{\frac{3x^2}{x^2}+\frac{2}{x^2}}$

$= \lim\limits_{x\to\infty} \dfrac{1+\frac{3}{x}}{3+\frac{2}{x^2}}$

$= \dfrac{1}{3}$

b) $3x^2-5x+1$

$\dfrac{d}{dx} = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$\lim\limits_{h\to 0} \dfrac{3(x+h)^2-5(x+h)+1-(3x^2-5x+1)}{h}$

$\lim\limits_{h\to 0} \dfrac{3x^2+6xh+h^2-5x-5h+1-3x^2+5x-1}{h}$ [illegible]

$\lim\limits_{h\to 0} \dfrac{6xh-5h+h^2}{h}$

$\lim\limits_{h\to 0} \dfrac{h(6x-5+h)}{h}$

$= \lim\limits_{h\to 0} 6x-5+h$

$= 6x-5$

Q2) a) $y = x^3+3x^2-1$

$\dfrac{dy}{dx} = 3x^2+6x$

b) $y = x\sqrt{x}$

$y = x^{3/2}$

$\dfrac{dy}{dx} = \frac{3}{2}x^{1/2}$

$= \dfrac{3\sqrt{x}}{2}$

c) $y = (x^2+3x)^4$

$\dfrac{dy}{dx} = 4(x^2+3x)^3(2x+3)$

d) $f(x) = \dfrac{x^2+1}{x-2}$

$f'(x) = \dfrac{(x-2)\times 2x - (x^2+1)\times 1}{(x-2)^2}$

$= \dfrac{2x^2-4x-x^2-1}{(x-2)^2}$

$= \dfrac{x^2-4x-1}{(x-2)^2}$

e) $f(x) = x\sqrt{x^2+1}$

$= x(x^2+1)^{1/2}$

$f'(x) = x.\frac{1}{2}(x^2+1)^{-1/2}2x + (x^2+1)^{1/2}$

$= \dfrac{x^2}{\sqrt{x^2+1}} + \sqrt{x^2+1}$

f) $y = \dfrac{3}{1-x\sqrt{2}}$

$= 3(1-x\sqrt{2})^{-1}$

$\dfrac{dy}{dx} = -3(1-x\sqrt{2})^{-2}\times -\sqrt{2}$

$= \dfrac{3\sqrt{2}}{(1-x\sqrt{2})^2}$

g) $y = \pi^3$

$\dfrac{dy}{dx} = 0$

h) $y = a^2x - ax^2$

$\dfrac{dy}{dx} = a^2 - 2ax$.

Q3) a) $f(x) = 4x^3-3x^2+2x+1$

$f'(x) = 12x^2-6x+2$

$f''(x) = 24x-6$

b) $y = (3x^2+2)^3$

$\dfrac{dy}{dx} = 3(3x^2+2)^2\times 6x$

$= 18x(3x^2+2)^2$

$\dfrac{d^2y}{dx^2} = 18x.2(3x^2+2).6x + 18(3x^2+$ [illegible]

$= 216x^2(3x^2+2) + 18(3x^2+2)^2$

$= 18(3x^2+2)(12x^2+(3x^2+2))$

$= 18(3x^2+2)(15x^2+2)$

Q4)

a) $y = x^2+2x-8$

$\dfrac{dy}{dx} = 2x+2$

increasing when $\dfrac{dy}{dx} > 0$

$2x+2 > 0$

$2x > -2$

$x > -1$

b) $y = x^3+x^2+x$

$y' = 3x^2+2x+1$

$y'' = 6x+2$

Concave down $y'' < 0$

$6x+2 < 0$

$x < -\frac{1}{3}$

c) $y = x^2-5x+6$

$y' = 2x-5$

gradient $= 1$

$2x-5 = 1$

$2x = 6$

$x = 3$.

$y = 9-15+6$

$= 0$

$\therefore$ point $(3,0)$

d) $y = x^3-3x^2-9x+1$

$\dfrac{dy}{dx} = 3x^2-6x-9$

gradient normal $= \frac{1}{9}$

$\therefore$ gradient tangent $= -9$

$\therefore 3x^2-6x-9 = -9$

$3x^2-6x = 0$

$3x(x-2) = 0$

$\therefore x = 0, \; 2$
$\;\;\;\; y = 1, \; -21$

$\therefore$ pts $(0,1)$ and $(2,-21)$

e) $y = x^3$

$\dfrac{dy}{dx} = 3x^2$

Slope $45° \Rightarrow$ gradient $= 1$

ie $3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\frac{1}{\sqrt{3}}$

$\therefore$ pts $\left(\frac{1}{\sqrt{3}}, \frac{1}{3\sqrt{3}}\right), \left(-\frac{1}{\sqrt{3}}, -\frac{1}{3\sqrt{3}}\right)$

Q5)

a) $y = x^3+ax^2-bx+c$

$\dfrac{dy}{dx} = 3x^2+2ax-b$

$\dfrac{d^2y}{dx^2} = 6x+2a$

pt. of inflexion $\dfrac{d^2y}{dx^2} = 0$ at $x = -\frac{1}{2}$.

$6(-\frac{1}{2})+2a = 0$

$-3+2a = 0$

$2a = 3$

$a = 1\frac{1}{2}$

$\therefore \dfrac{dy}{dx} = 3x^2+3x-b$

S.P. at $x = -2$. $\dfrac{dy}{dx} = 0$

$12-6-b = 0$

$b = 6$

$\therefore y = x^3+\dfrac{3x^2}{2}-6x+c$

x intercept at $x = 1$: $y = 0$

$1+1\frac{1}{2}-6+c = 0$

$c = 3\frac{1}{2}$

b) $y = a(x+b)^2-8$

$\dfrac{dy}{dx} = 2a(x+b)$

when $x = 4$, $\dfrac{dy}{dx} = 2$.

$2a(4+b) = 2$

$8a+2ab = 2$

$4a+ab = 1$ --- (1)

$x = 4$, $y = 8$

$8 = a(4+b)^2-8$

$a(4+b)^2 = 16$ --- (2)

From (1) $a = \dfrac{1}{4+b}$.

Sub into (2)

$\dfrac{1}{4+b}(4+b)^2 = 16$

$(4+b)^2 = 16(4+b)$

$16+8b+b^2 = 64+16b$

$b^2-8b-48 = 0$

$(b-12)(b+4) = 0$

$b = 12, -4$.
$a = \frac{1}{16}$ → no Solution

$\therefore a = \frac{1}{16}, b = 12$.

c) $y = \sqrt{2x+4}$

$y = (2x+4)^{1/2}$

$\dfrac{dy}{dx} = \frac{1}{2}(2x+4)^{-1/2}\times 2$

$= \dfrac{1}{\sqrt{2x+4}}$

$\therefore y.\dfrac{dy}{dx}$

$= \sqrt{2x+4}.\dfrac{1}{\sqrt{2x+4}}$

$= 1$ (a constant)

d) $f(x) = x^3-2x+1$

$f'(x) = 3x^2-2$

$f(-1) - f'(-1)$

$= (-1+2+1) - (3-2)$

$= 2-1$

$= 1$

e) $f(x) = x^2-3x+3$

$f'(x) = 2x-3$

$f(b) = b^2-3b+3$

$f'(b) = 2b-3$.

$f(b) = f'(b)$

$b^2-3b+3 = 2b-3$

$b^2-5b+6 = 0$

$(b-3)(b-2) = 0$

$b = 2, 3$.

Q6) a) $y = 3x^3-8x^2$

$\dfrac{dy}{dx} = 9x^2-16x$

at $x = 2$: $\dfrac{dy}{dx} = 4$.

$\therefore$ eqn. tangent

$y - -8 = 4(x-2)$

$y+8 = 4x-8$

$y = 4x-16$

b) $y - -8 = -\frac{1}{4}(x-2)$

$4y+32 = -x+2$

$x+4y+30 = 0$

c) A: $(y=0)$ $0 = 4x-16$

$x = 4$

$\therefore$ A $(4,0)$

B: $(x=0)$: $4y+30 = 0$

$y = -7\frac{1}{2}$

$\therefore$ B $(0, -7\frac{1}{2})$

d) $AB = \sqrt{4^2+(-7\frac{1}{2})^2}$

$= \sqrt{16+56.25}$

$= \sqrt{\frac{289}{4}}$

$= \dfrac{17}{2}$.

Q7) a) i $y = 1+3x-x^3$

$\frac{dy}{dx} = 3-3x^2$

$\frac{d^2y}{dx^2} = -6x$

turning pts. $\frac{dy}{dx}=0$

$3-3x^2=0$

$1-x^2=0$

$(1-x)(1+x)=0$

$x = 1, -1$

$y = 3, -1$

$F''(1)<0$ ∴ $(1,3)$ max; $F''(-1)>0$ ∴ $(-1,-1)$ min

ii) pts. of inflection $\frac{d^2y}{dx^2}=0$

$-6x=0$

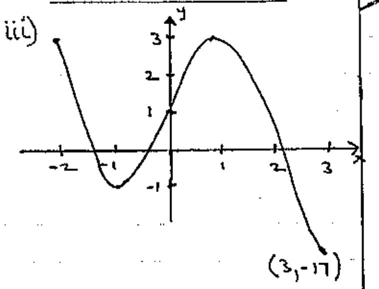
$x=0.$

$F''(-1)>0,\ F''(1)<0$

∴ change in concavity

∴ pt. of inflexion $(0,1)$

iii)



v) min: -17

max: 3.

$y = x + \frac{1}{x}$

i) $x = 0$

ii) as $x\to\infty$ $\frac{1}{x}\to 0$

∴ $y \to x$.

iii) $y = x + \frac{1}{x}$

$\frac{dy}{dx} = 1 - \frac{1}{x^2}$

$\frac{d^2y}{dx^2} = \frac{2}{x^3}$

turning points $\frac{dy}{dx}=0$

$1-\frac{1}{x^2}=0$

$1 = \frac{1}{x^2}$

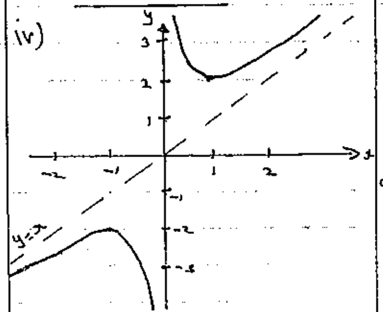
$x^2 = 1$

$x = 1, -1$

$y = 2, -2$

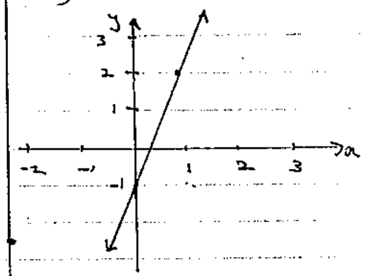
$F''(1)>0$ ∴ $(1,2)$ min; $F''(-1)<0$ ∴ $(-1,-2)$ max

iv)

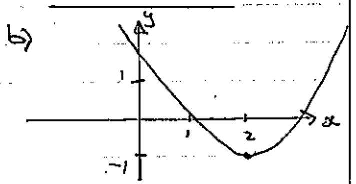


Q8)

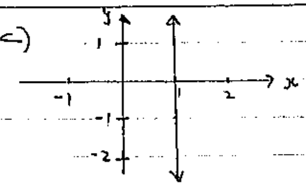
a) $\frac{dy}{dx} = 3.$



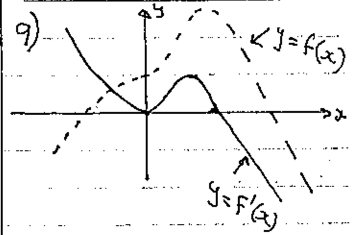
b)



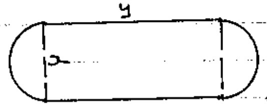
c)



9)



10)



a) $2y + \pi x = 1000$

$2y = 1000 - \pi x$

$y = 500 - \frac{\pi x}{2}$

b) $A = xy + \pi\left(\frac{x}{2}\right)^2$

$= x\left(500 - \frac{\pi x}{2}\right) + \frac{\pi x^2}{4}$

$= 500x - \frac{\pi x^2}{2} + \frac{\pi x^2}{4}$

$= \frac{2000x - 2\pi x^2 + \pi x^2}{4}$

$= \frac{2000x - \pi x^2}{4}$

c) $A = \frac{2000x - \pi x^2}{4}$

$\frac{dA}{dx} = \frac{2000 - 2\pi x}{4}$

$= \frac{1000-\pi x}{2}$

$\frac{d^2A}{dx^2} = \frac{-\pi}{2}$

max when $\frac{dA}{dx} = 0$

$\frac{1000-\pi x}{2} = 0$

$1000 - \pi x = 0$

$x = \frac{1000}{\pi}$

$A = \frac{2000x - \pi x^2}{4}$

$= \frac{2000\times\frac{1000}{\pi} - \pi\left(\frac{1000}{\pi}\right)^2}{4}$

$= \frac{2000\,000 - 1000\,000}{4\pi}$

$= \frac{250\,000}{\pi}$ m²

$= \frac{25}{\pi}$ hectares

≈ 8 hectares.

PART B.

Q1) a) i) centre $(0,0)$

radius $= 4$

ii) centre $(-1,2)$

radius $= 2\sqrt{3}$

iii) $x^2-4x+y^2+6y+6=0$

$x^2-4x+4+y^2+6y+9 = -6+4+9$

$(x-2)^2+(y+3)^2 = 7$

radius $= \sqrt{7}$

centre $(2,-3)$

b) $y = x$ or $y = -x$

c) $P(x,y)$, $A(4,0)$ $B(1,0)$

$PA = 2PB$

$\sqrt{(x-4)^2+y^2} = 2\sqrt{(x-1)^2+y^2}$

$x^2-8x+16+y^2 = 4(x^2-2x+1+y^2)$

$x^2-8x+16+y^2 = 4x^2-8x+4+4y^2$

$3x^2+3y^2 = 12$

$x^2+y^2 = 4$

d) $A(3,0)$, $B(-3,0)$ $P(x,y)$

$m_{PA} \cdot m_{PB} = -1$

$\frac{y}{x-3}\times\frac{y}{x+3} = -1$

$\frac{y^2}{x^2-9} = -1$

$y^2 = 9-x^2$

$x^2+y^2 = 9.$

Q2) a) i) 4

ii) $(0,-4)$

iii) $y = 4$

b) $x^2+6x-33 = 12y$

$x^2+6x = 12y+33$

$x^2+6x+9 = 12y+33+9$

$(x+3)^2 = 12y+42$

$(x+3)^2 = 12(y+3\tfrac{1}{2})$

i) $(-3, -3\tfrac{1}{2})$

ii) $(-3, -\tfrac{1}{2})$

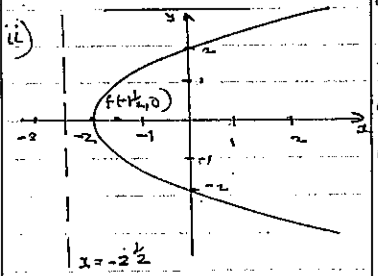
iii) $y = -6\tfrac{1}{2}.$

c) $2y^2 = 4x+8$

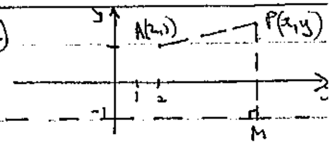
i) $y^2 = 2x+4$

$(y-0)^2 = 2(x+2)$

ii)



d)



$PA = PM$

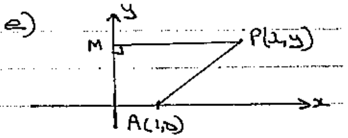
$\sqrt{(x-2)^2+(y-1)^2} = y+1$

$x^2-4x+4+y^2-2y+1 = y^2+2y+1$

$x^2-4x+4 = 4y$

$(x-2)^2 = 4y$

e)



$PA = PM$

$\sqrt{(x-1)^2+y^2} = x$

$x^2-2x+1+y^2 = x^2$

$y^2 = 2x-1$

$y^2 = 2(x-\tfrac{1}{2})$

f) i) $x^2 = 20y.$

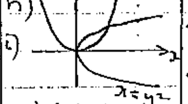
ii) $y^2 = 8x$

g) eqn of the form: $(x-1)^2 = 4a(y-4)$

passes through $(3,5)$: $2^2 = 4a(1)$

∴ $a = 1$. ∴ eqn: $(x-1)^2 = 4(y-4)$

h) i)



Test $(0,0)$, $(1,1)$ in each equation.

iii) $y = x^2$

$\frac{dy}{dx} = 2x$

at $x=1$ $\frac{dy}{dx} = 2$

$y-1 = 2(x-1)$

$y = 2x-1$

$P(0,-1)$

iv) $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

at $x=1$ $\frac{dy}{dx} = \tfrac{1}{2}$

$y-1 = \tfrac{1}{2}(x-1)$

$2y = x+1$ ∴ $Q(-1,0)$

v) $AP = \sqrt{(1-0)^2+(1+1)^2}$

$= \sqrt{5}$

$AQ = \sqrt{(1+1)^2+1^2}$

$= \sqrt{5}$